VARIATIONAL SOLUTIONS TO E-PLANE BILATERAL SEPTUM IN WAVEGUIDE

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ABSTRACT

The variational technique has been extended to the study of the E-plane touching and nontouching bilateral septum in waveguide. Computed data using the developed equations show good agreement with the experimental data. Closed form expressions of input admittance and equivalent circuit are obtained and can be directly used in practice to design E-plane circuits.

INTRODUCTION

E-plane circuits have found many applications in microwave and millimeter-wave circuits. One of the key elements is the bilateral septum in waveguide. The attractiveness of using bilateral septum compared to unilateral septum is that a higher Q factor can be achieved. Thus, this aspect could be a crucial consideration in the low loss circuit design. Compact filters, planar diode mounts, and matching networks in waveguide can be constructed at a minimum cost provided that no extensive tuning are involved. To achieve the above goal, an accurate characterization of the E-plane bilateral septum is required. Techniques based on mode matching, complex residue calculus, and field expansion had been applied to study bilateral septum [1-4]. However, these techniques have a drawback because they are not being straight forward due to the required usage of complex matrix manipulations. In addition, the final equations are in numerical form. In this paper, a variational technique [5] utilizing the extrimization process is used to study the E-plane bilateral septum. Both touching and nontouching septum cases are considered in the analysis. The equations for input scattering parameters and equivalent circuits are written in closed form so that it can be used in practice. The design of E-plane circuits using these elements is consequently simplified because it alleviates the empirical determination of the element value.

FORMULATION

Fig. 1 shows the geometries of the E-plane bilateral septum to be analyzed. The bilateral septum is composed of two metal strips (length y_1) inserted into a waveguide with a and b dimensions. The assumed thin strips are mounted at x_1 and x_2 in the transverse direction. The strips have the same width with $w_1 = w_2 = w$.

The total field at any given point r inside the waveguide can be related to its corresponding dominant mode incident field and the scattered electric field

$$E_t = \phi_1(x)\, e^{-\beta_{10} z} - j\omega\mu \int_V \frac{(2-\delta_m)(k_o^2 - k_{ym}^2)}{ab k_o^2 \beta_{nm}} e^{-\beta_{nm}[z-z']} \phi_n(x)\phi_n(x') \cos(k_{ym} y)\cos(k_{ym} y') \left[J^I(x',y',z') + J^{II}(x',y',z') \right] dx',dy',dz' \quad (1)$$

$J^I(x',y',z')$ and $J^{II}(x',y',z')$ are the current of the two metal strips. β_{nm} is the propagation constant, k_{ym} is the cut off wave number in the y direction, and k_o is the medium wave number. $\delta_m = 0$ for m = 0 and $\delta_m = 1$ otherwise.

The boundary condition requires the total tangential electric field to vanish on the metal surface. When $E_t = 0$, the dominant mode reflection coefficient T at z = 0 is

$$T = \frac{-j\omega\mu}{\beta_{10}} \int_V e^{-\beta_{10} z'} \phi_n(x') \left[J^I(x',y',z') + J^{II}(x',y',z') \right] dx',dy',dz' \quad (2)$$

Multiplying both sides of eqn. (1) by $(J^I(x,y,z) + J^{II}(x,y,z))$. Substituting eqn. (2) into the left hand side of the new eqn. (1) and using the relation $Y_{in} = (1 - T)/(1 + T)$, a generalized equation for normalized input admittance is now derived,

$$Y_{in} = \frac{\sum_{n=1}^{\infty} \sum_{m=0}^{\infty} h_1 \alpha_{nm} \gamma_{nm} + (\alpha_{10}/\beta_{10})}{\sum_{n=1}^{\infty} \sum_{m=0}^{\infty} h_1 \alpha_{nm} \gamma_{nm} - (\alpha_{10}/\beta_{10})} \quad (3a)$$

where

$$h_1 = \frac{(2-\delta_m)(k_o^2 - k_{ym}^2)}{k_o^2 \beta_{nm}} \quad (3b)$$

$$\alpha_{nm} = \int_V e^{-\beta_{nm} z} \phi_n(x) \cos(k_{ym} y) \left[J^I(x,y,z) + J^{II}(x,y,z) \right] dxdydz \quad (3c)$$

$$\gamma_{nm} = \int_V e^{-\beta_{nm} z'} \phi_n(x') \cos(k_{ym} y') \left[J^I(x',y',z') + J^{II}(x',y',z') \right] dx'dy'dz' \quad (3d)$$

Equations for α_{nm} and γ_{nm} are evaluated according to the configurations of the bilateral septum.

Nontouching Bilateral Septum:

The current distribution of the nontouching septum is assumed to have a form

$$J^I(x,y,z) + J^{II}(x,y,z) = \sin(k_o(y - b + y_1))\, e^{-\beta_{10} z} \left[\delta(x-x_1) + R\,\delta(x-x_2) \right] \quad (4)$$

The current is maximum at the contact point of the septum and the waveguide wall and it decreases sinusoidally until it becomes zero at the open end of the strip. The direction of the current in the second strip is the same to that in the first strip. Evaluating eqns. (3c) and (3d) through the use of eqn. (4) over the boundary conditions leads to an expression for Y_{in},

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$$Y_{in} = \frac{\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} h_1\, C_{nm}\,[A_{nm}^2 + R^2 B_{nm}^2 + 2R\, A_{nm}B_{nm}] + (D_{10}^2/\beta 10)[A_{10} + R\, B_{10}]^2}{\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} h_1\, C_{nm}\,[A_{nm}^2 + R^2 B_{nm}^2 + 2R\, A_{nm}B_{nm}] - (D_{10}^2/\beta 10)[A_{10} + R\, B_{10}]^2} \tag{5a}$$

where

$$A_{nm} = \frac{k_o}{k_o^2 - k_{ym}^2}\sin(k_{nx}x_1)\cos(m\pi)\,[\cos(k_{ym}y_1) - \cos(k_o y_1)] \tag{5b}$$

$$B_{nm} = \frac{k_o}{k_o^2 - k_{ym}^2}\sin(k_{nx}x_2)\cos(m\pi)\,[\cos(k_{ym}y_1) - \cos(k_o y_1)] \tag{5c}$$

$$Cnm = \frac{1}{\beta 10^2 + \beta nm^2}\left[\frac{\beta 10 - \beta nm}{\beta 10} - 2e^{-w(\beta 10 + \beta nm)} + \frac{\beta 10 + \beta nm}{\beta 10} e^{-2w\beta 10}\right] \tag{5d}$$

$$D_{10} = \frac{1}{2\beta 10}[1 - e^{-2w\beta 10}] \tag{5e}$$

A_{nm}, B_{nm}, and C_{nm} are the coefficients at mode index nm. R is defined as the current density ratio between the two distributions existing on the strips and can be computed through the extrimization process by setting $d(Y_{in})/df = 0$.

$$R = \frac{\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} A_{10}A_{nm}B_{nm} - B_{10}A_{nm}^2}{\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} B_{10}A_{nm}B_{nm} - A_{10}B_{nm}^2} \tag{6}$$

Touching Bilateral Septum:

The same procedure is carried out for the touching septum as in the previous case. Except for the variation in the y direction, all other terms in the current distribution of the touching septum are identical to the nontouching case. Setting the term $\sin(k_o(y-b+y_1))$ in eqn. (4) to unity and evaluating eqns. (3c) and (3d), a closed form for Y_{in} is obtained and is written as follows:

$$Y_{in} = \frac{\sum_{n=1,\ \text{at } m=0}^{\infty} \frac{C_{no}}{\beta no}[A_{no}^2 + R^2 B_{no}^2 + 2RA_{no}B_{no}] + \frac{D_{10}^2}{\beta 10}[A_{10} + R\, B_{10}]^2}{\sum_{n=1,\ \text{at } m=0}^{\infty} \frac{C_{no}}{\beta no}[A_{no}^2 + R^2 B_{no}^2 + 2RA_{no}B_{no}] - \frac{D_{10}^2}{\beta 10}[A_{10} + R\, B_{10}]^2} \tag{7a}$$

with

$$A_{no} = \sin(k_{nx}x_1) \tag{7b}$$

$$B_{no} = \sin(k_{nx}x_2) \tag{7c}$$

$$R = \frac{\sum_{n=1}^{\infty} A_{no}B_{no}A_{10} - A_{no}^2 B_{10}}{\sum_{n=1}^{\infty} A_{no}B_{no}B_{10} - B_{no}^2 A_{10}} \tag{7d}$$

C_{no} and D_{10} are eqns. (5d) and (5e). Note that in this case the total input admittance is just a summation of the modal admittance with index m = 0. Modal admittance is zero for $m \geq 1$.

The T-equivalent circuit for the bilateral septum can be expressed in Fig. (1c). After Y_{in} is obtained, the series reactance X_a and shunt reactance X_b can be now computed

$$X_a = -X_b \pm \frac{-B_{in}/(G_{in}^2 + B_{in}^2)}{(G_{in}/(G_{in}^2 + B_{in}^2)) - 1} \tag{8a}$$

$$X_b = \pm\left\{(G_{in}/(B_{in} + B_{in}))\left[1 + \frac{-B_{in}/(G_{in}^2 + B_{in}^2)}{G_{in}/(G_{in}^2 + B_{in}^2)}\right]\right\}^{1/2} \tag{8b}$$

The signs in eqn. (8) is determined using the Foster reactance theorem in which $d(X)/df > 0$.

VERIFICATION

The calculated data of both touching and nontouching bilateral septum were verified experimentally. Data are obtained with a = 16.12 mm, b = 8.10 mm, x_1 = 6.497 mm, x_2 = 9.623 mm, and w = 0.5 mm. Large values of m and n are chosen so as to ensure the covergence of the input admittance. Fig. (2a) illustrates the normalized susceptance versus frequency ranging from 12.0 Ghz to 18.0 Ghz for y_1 = 6.51 mm. Fig. (2b) shows the frequency response when the septum touched the bottom waveguide wall. In the former case, the susceptance changes sign from positive to negative at the resonance frequency after magnitude reaches maximum value. In the latter case, the susceptance remains capacitive throughout the frequency band. Data obtained using developed equations are also presented for comparison and show reasonable agreement with the measured data.

CONCLUSION

A theoretical analysis that makes it possible to compute the input scattering parameters and its T-equivalent circuit for the E-plane touching and nontouching bilateral septum, has been presented and verified experimentally. A major advantage of using variational method compared to the other theoretical techniques is that the final equations are expressed in closed form and hence it can be used directly to design E-plane circuits.

REFERENCES

[1] Y.C. Shih, T. Itoh, and L. Bui, "Computer Aided Design of Millimeter Wave Filters," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-31, Feb. 1983, pp. 135-142.

[2] Y. C. Shih and T. Itoh, "Design of Waveguide E-plane Filter with All Metal Inserts," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-32, July 1984, pp. 695-704.

[3] F. Arndt, J. Bornemann, D. Grauerholz, and R. Vahldieck,"Low Insertion Loss Fin Line Filters for Millimeter Wave Applications, " Proc. 11th European Microwave Conf., 1981, pp. 309-314.

[4] J. Bornemann, F. Arndt, R. Vahldieck, and D. Grauerholz, "Double Planar Integrated Millimeter Wave Filter," Proc. 13th European Microwave Conf., 1983, pp. 168-173.

[5] K. Chang and P. Khan, "Equivalent Circuit of a Narrow Axial Strip in Waveguide," IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-24, Sept. 1976, pp. 611-614.

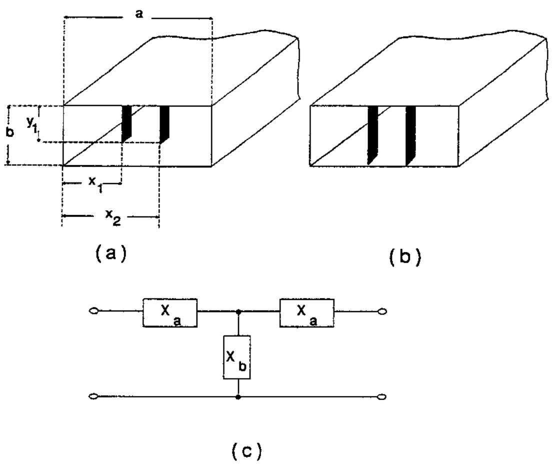
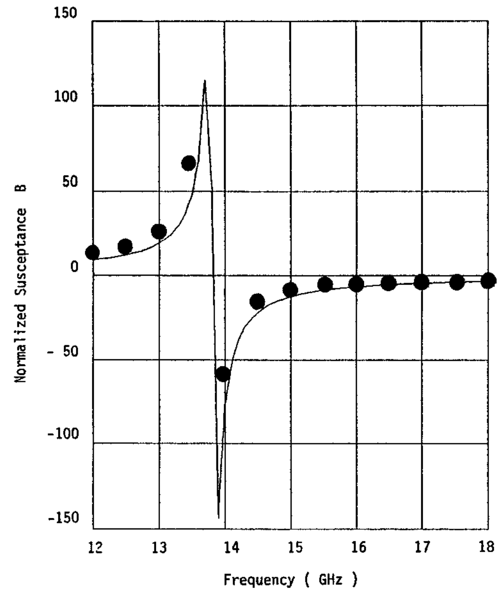


Fig. 1. Geometry of the E-plane (a) nontouching and (b) touching bilateral septum in waveguide. (c) Its corresponding T-equivalent circuit.



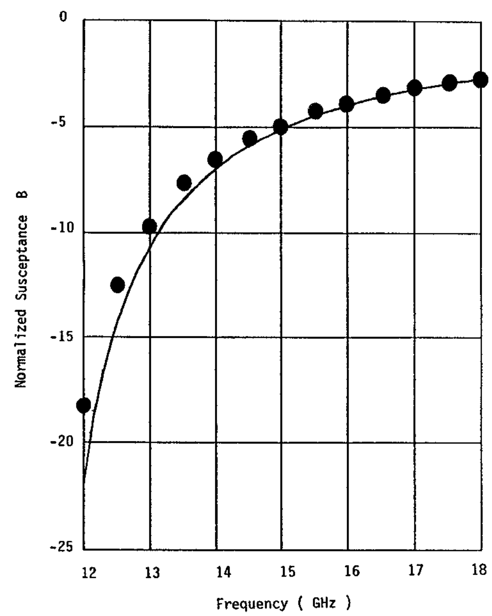


Fig. 2. Normalized input susceptance of the bilateral septum in Ku-band waveguide with a =16.12 mm, b = 8.10 mm, x_1 = 6.497 mm, x_2 = 9.623 mm, and w = 0.5 mm. (a) Nontouching case with y_1 = 6.51 mm. (b) Touching case with y_1 = 8.10 mm. ———— Computed data using this technique. ● ●●● Measured data.